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Abstract

Many acceleration-sensitive, microgravity science experiments will require active vibration isolation from the manned orbiters on which they will be mounted. The isolation problem, especially in the case of a tethered payload, is a complex three-dimensional one that is best suited to modern-control design methods. These methods, although more powerful than their classical counterparts, can nonetheless go only so far in meeting the design requirements for practical systems. Once a tentative controller design is available, it must still be evaluated to determine whether or not it is fully acceptable, and to compare it with other possible design candidates. Realistically, such evaluation will be an inherent part of a necessarily iterative design process. In this paper, an approach is presented for applying complex μ-analysis methods to a closed-loop vibration isolation system (experiment plus controller). An analysis framework is presented for evaluating nominal stability, nominal performance, robust stability, and robust performance of active microgravity isolation systems, with emphasis on the effective use of μ-analysis methods.

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A new technique is also included for deriving guarantees on allowable umbilical and payload mass variations, using complex μ-analysis.

Introduction

The microgravity vibration isolation problem has received considerable attention in recent years. A number of materials processes and fluid physics science experiments, planned for study in a "weightless" space environment, have run into unacceptably high background acceleration levels [1]. The low-frequency disturbances of most concern are a natural accompaniment of space-flight, with its large, flexible, unloaded structures and random, human-induced excitations. The combined need, with many experiments, for human interaction and for umbilicals connecting orbiter with payload, has resulted in a very difficult, three-dimensional, active-isolation design problem.

Recent work by the authors has produced an extended H_2 synthesis framework, along with an associated general design philosophy, for developing a robust microgravity vibration-isolation controller [2]. Other approaches (*e.g.*, classical design or μ synthesis) are, of course, also theoretically capable of producing acceptable designs. But no approach can guarantee a satisfactory controller, short of an iterative design-and-analysis procedure. Analysis of any controller candidate is necessary to ensure that it, in fact, meets stability and performance requirements for a reasonable degree of model uncertainty. μ-analysis methods can provide conservative guarantees of such acceptability [3]. μ analysis also allows the engineer to assign a relative merit to each competing controller design.

This paper seeks to provide an analysis framework and philosophy for evaluating a given isolation controller candidate, with emphasis on the effective use of μ-analysis methods. Particular stress is placed on evaluating the nominal stability, nominal performance, robust stability, and robust performance of the associated closed-loop isolation system. The work builds on the microgravity control synthesis framework proposed by the authors in [4].

A Basic Isolation System

A generic microgravity vibration isolation system is depicted below in Fig. 1. A payload, such as a microgravity science experiment, is acted upon by actuators (typically noncontacting, *e.g.*, Lorentz or electromagnetic) that are commanded by a control system. This control system uses measurements, such as payload positions and accelerations, to develop the control signals, typically currents or voltages. The objective of controller synthesis is to develop a stable, robust controller that meets or exceeds the performance requirements. A framework for developing such a controller, by extended H_2 synthesis, has been presented in a previous paper [4]. For the purposes of the present work, it is assumed that a tentative controller has been synthesized and is in need of evaluation.

Fig. 2. shows a transfer-function block diagram of the closed-loop system in state-space form. This is the basic model used for closed-loop system analysis. The plant $\{A,B,C,D\}$ (*i.e.*, payload plus umbilical plus actuator) is subject to direct and indirect disturbances; both kinds are included in input disturbance vector $\underline{f}_s$. The direct disturbances are those which act directly upon the payload; for example, these could be caused by air currents, astronaut contact, the flow of fluids for lubrication or cooling, or rotating machinery mounted on the experiment platform. The indirect disturbances act upon the payload through the umbilical. Due to the umbilical stiffness, any motion of the space platform wall relative to the experiment will cause such a disturbance.

Some of the plant states $\underline{x}$ (not shown) are inaccessible; a subset $\underline{y}$ represents the ideal measurements, uncontaminated by sensor noise. Measurement vector $\underline{z}$ represents the actual measurements, which are contaminated by output sensor-noise vector $\underline{f}_n$. The synthesized controller, to be evaluated, has state-space form $\{A_{F/B},B_{F/B},C_{F/B},D_{F/B}\}$. E_s and E_n are simply selection matrices, and the control vector is $\underline{u}$.

Controller Evaluation

Once a microgravity vibration isolation controller has been synthesized, it must be evaluated for acceptability and relative merit. One major design consideration is simplicity; the controller's state-space form should have no more states than necessary. For an analog implementation, controller complexity translates directly into hardware complexity; for a digital controller, the cost is largely in terms of computational power and speed. A controller which is robust in stability and performance but which is too complicated for employment is of no use. Modern control methods, especially with the augmentation accompanying frequency weighting and disturbance accommodation, can result in controllers that have an unnecessarily large number of states. The controller dimensionality can usually be reduced substantially by employing modal reduction and/or the balance-and-truncate technique ("Moore's method;" [5], 1981). And even if controller reduction is not deemed necessary, it is advisable for the sake of design simplicity.

Another important consideration is controller stability. Although unstable controllers are used on occasion (provided the closed-loop system is stable) and need not be rejected out-of-hand, they cannot easily be bench-tested. An ideal controller designed by extended H_2- or μ synthesis is guaranteed to stabilize the nominal plant; but it is not guaranteed to be open-loop stable, since some plants can only be stabilized by unstable controllers. Application of reduction techniques can also lead to controller destabilization. It is best to conduct an eigenvalue check of any controller candidate's "A-matrix" (*i.e.*, its dynamic matrix) to ensure controller stability.

Once the controller model has been checked for simplicity and stability, it is ready for attachment to the model of the plant. There are four crucial checks that must now be made of the closed loop system; *viz.*, nominal stability, nominal performance, robust stability, and robust performance [3]. The nominal analysis model (Fig. 2) is the conceptual starting point for conducting these checks, each of which is treated individually below.

Nominal Stability

An unstable closed-loop system cannot, of course, provide the isolation desired. Stability is the most basic system requirement for any microgravity vibration isolation system. The extended H_2-synthesis method, when used for controller design, provides an inherent guarantee of stability for the nominal plant with full-state feedback. However, H_2 synthesis occasionally produces an unstable closed-loop system due to numerical anomalies. So closed-loop system eigenvalues should still be checked. The same is true with a controller designed by μ synthesis.

The well-known "separation principle" guarantees that for a perfectly known plant, a stable asymptotic (*i.e.*, Luenberger) observer will not destabilize the system. Thus, with the full-order observer (*i.e.*, observing all states and pseudostates), nominal closed-loop stability is assured. However, models of dynamic systems are never perfect, so nominal stability is no guarantee of actual isolation-system stability; and the use of an umbilical for the present problem makes precise modeling especially difficult. Still, a check of nominal stability certainly provides a reasonable starting point.

If the order of the feedback controller is reduced, the guarantee of nominal closed-loop stability provided by extended H_2 synthesis is lost. But simple checks of the closed-loop-system eigenvalues (*i.e.*, the eigenvalues of the closed-loop system A-matrix) can readily verify (or contraindicate) stability, regardless of the design methodology used. For the system shown in Fig. 2, the closed-loop-system A-matrix (from input $\underline{u}$ to output $\underline{z}$, for simplicity, and without loss of generality) is

$$A_{CL} = \begin{bmatrix} A + B(I - D_{F/B}D)^{-1} D_{F/B}C & B(I - D_{F/B}D)^{-1} C_{F/B} \\ B_{F/B}(I - D D_{F/B})^{-1} C & A_{F/B} + B_{F/B}(I - D D_{F/B})^{-1} D C_{F/B} \end{bmatrix} \tag{1}$$

where $\left[\begin{array}{c|c} A_{F/B} & B_{F/B} \\ \hline C_{F/B} & D_{F/B} \end{array}\right]$ represents the transfer function matrix in the feedback path from $\underline{z}$ to $\underline{u}$.

Nominal Performance

A potential controller design for a microgravity vibration-isolation system must do more than provide stability; it must also give acceptable system performance when its model is attached to the nominal plant model. Orbiter motions and sensor noise must both be rejected to specified levels. And the necessary control signals must be small enough to prevent actuator- or amplifier saturation. Using the various transfer functions represented by the nominal analysis model, the designer can verify that the nominal controlled plant meets such design specifications. Since system uncertainties (*e.g.*, umbilical- or actuator model inaccuracies) will degrade actual closed-loop system performance, it is generally desirable to exceed the performance specifications during controller design, so that as large a set of off-nominal plants as possible (or desirable) will still meet the design specifications. Robust performance will be checked at a later stage; but it should be kept in mind here that a nominal performance which is only marginal will probably be unacceptable when model inaccuracies are taken into account. The goal, of course, is for the isolation system to meet the performance requirements in hardware.

The uncontrolled plant for the microgravity vibration-isolation problem has a strictly proper transfer-function matrix (*i.e.*, $D \equiv O$, the zero matrix). Accordingly, the closed-loop transfer functions from process-noise inputs $\underline{f}_S$ to measurements $\underline{z}$ are

$$H^{CL}_{\underline{z},\underline{f}_s} = \left[\begin{array}{cc|c} A + B D_{F/B} C & B C_{F/B} & I \\ B_{F/B} C & A_{F/B} & O \\ \hline E_s C & O & O \end{array}\right] \tag{2a}$$

And from sensor noise inputs $\underline{f}_n$ to measurements $\underline{z}$, the closed-loop transfer functions are

$$H^{CL}_{\underline{z},\underline{f}_n} = \left[\begin{array}{cc|c} A + B D_{F/B} C & B C_{F/B} & B D_{F/B} \\ B_{F/B} C & A_{F/B} & B_{F/B} \\ \hline C & O & I \end{array}\right] \tag{2b}$$

The closed-loop system must achieve acceptable isolation, but it must do so without unacceptable sensitivity to sensor noise. These two tasks involve an inherent trade-off. High loop gains are necessary to reject input disturbances at a given frequency. These correspond to small transmissibility gains, *i.e.*, to small magnitudes of the complementary sensitivity-function matrix T. On the other hand, low loop gains are necessary to reject sensor noise. These correspond to small magnitudes of the sensitivity-function matrix S. Since the sum of T and S is the identity matrix I [6], the designer cannot achieve arbitrarily low sensitivity to noise in the same frequency band where good disturbance rejection is to be accomplished. This unavoidable trade-off can be measured by use of the above two transfer-function matrices.

The system must also employ a control signal that does not become excessive for realistic disturbance levels. If amplifiers or actuators are allowed to saturate, the resulting nonlinearities will greatly complicate both the synthesis and the analysis problems. The following transfer-function matrix provides a measure of this aspect of nominal system performance.

$$H^{CL}_{\underline{u},\underline{f}_s} = \left[\begin{array}{cc|c} A + B\,D_{F/B}C & BC_{F/B} & E_S \\ B_{F/B}C & A_{F/B} & O \\ \hline D_{F/B}C & C_{F/B} & O \end{array}\right] \tag{2c}$$

Simple Form for Nominal H_2 Controller

For a controller designed by extended H_2 synthesis [4], the controller transfer-function matrix $\left[\begin{array}{c|c} A_{F/B} & B_{F/B} \\ \hline C_{F/B} & D_{F/B} \end{array}\right]$ has a particularly convenient form. First, define $n := \dim(\underline{x})$, the number of states in the plant model; and let the disturbance-accommodation pseudostate vector be $\underline{\xi} := \left[\underline{\xi}_1^T, \underline{\xi}_2^T\right]^T$, with $\underline{\xi}_1$ and $\underline{\xi}_2$ corresponding to input- and output disturbances, respectively. Assume the use of an asymptotic observer for state reconstruction. When all states and pseudostates are reconstructed in the observer, the full observer-synthesis model is used (Fig. 3a). In this case,

$$\left[\begin{array}{c|c} A_{F/B} & B_{F/B} \\ \hline C_{F/B} & D_{F/B} \end{array}\right] = \left[\begin{array}{c|c} {}^1A - {}^1B\,{}^1K - {}^1L\,{}^1C & {}^1L \\ \hline -{}^1K & O \end{array}\right] \tag{3}$$

where K is the optimal feedback gain matrix, and L is the observer gain matrix. The pre-superscripts "*1*" reflect the appropriate augmentation.

The frequency-weighting pseudostates need not be reconstructed in the observer, since they can be produced from the state observations and control signals, by simple filtering. One can then (more typically) use an observer-synthesis model of reduced order (Fig. 3b), in which case

$$\left[\begin{array}{c|c} A_{F/B} & B_{F/B} \\ \hline C_{F/B} & D_{F/B} \end{array}\right] = \left[\begin{array}{c|c} {}^{1}A - {}^{1}B\,{}^{1}K - {}^{2}L_{s}\,{}^{1}C & {}^{2}L_{s} \\ \hline -{}^{1}K & O \end{array}\right] \tag{4a}$$

where ${}^{2}L_{s} := \begin{bmatrix} L_x \\ O \\ O \\ L_\xi \end{bmatrix}$, for ${}^{2}L := \begin{bmatrix} L_x \\ L_\xi \end{bmatrix}$, with L_x consisting of the first n rows of ${}^{2}L$. (4b,c)

The pre-superscript "2" indicates that the frequency-weighting pseudostates were omitted in the augmented plant model used for determining the observer gains [4]. Eqs. (3) and (4) can be used with Eqs. (1) and (2) to evaluate system nominal stability and -performance with relative ease.

Robust Stability

To be practical, a vibration-isolation system must be stable not only for the nominal plant but for a range of off-nominal plants as well. For the microgravity isolation problem, the payload mass will in most cases be known quite accurately. But fluid motion (*e.g.*, for cooling or in studies of liquid bridges, surface-tension driven convection, or encapsulated crystal growth) can cause actual or apparent mass/inertia variations. And umbilical and actuator nonlinearities will typically cause the respective models to be quite inaccurate. A controller design must take into account such effects, so that system stability can be ensured for at least the anticipated range of model uncertainties. A closed-loop-system of this character is said to have "robust stability."

In classical control theory, the gain margin (GM) and phase margin (PM) of a single-input-single-output (SISO) system are the familiar measures of stability robustness. They measure, respectively, the amount of gain or phase that can be inserted into the feedback loop of a transfer-function block diagram without leading to system instability. (This may be conceptualized as the turning of a "gain knob" or "phase knob," respectively, until instability is encountered.) Accordingly, they provide the analyst with separate measures of allowable uncertainty in loop gain or phase.

If the loop gain and phase are allowed to vary simultaneously, this uncertainty can be represented correspondingly by a complex gain $e^{\alpha+j\beta} = re^{j\beta}$. Let $\Delta(s)$ be a complex "ball" representing all possible complex gains of some magnitude r, and let $M(s)$ be the transfer function of the (stable closed-loop) system "seen" by these gains. (Refer to Fig. 4.) The magnitude of the scalar product $\Delta(j\omega)M(j\omega)$ must be less than one to guarantee that the uncertainty does not destabilize the system. Consequently, the magnitude of the inverse of $M(j\omega)$ $\left(i.e., \left|M^{-1}(j\omega)\right|\right)$ provides a measure of the magnitude of complex uncertainty allowable in the loop, as a function of frequency ω. The minimum of the plot of $\left|M^{-1}(j\omega)\right|$ provides a guarantee of allowable uncertainty size for all frequencies.

For multiple-input-multiple-output (MIMO) systems, a robustness analysis method very similar to the Δ-complex-gain method described above can be used. In this analysis, $M(s)$ is a transfer-function matrix and $\Delta(s)$ is a transfer-function matrix of complex gains representing uncertainty in the system. The maximum singular value of the matrix product $\Delta(j\omega)M(j\omega)$ must be less than one to ensure a stable system. Consequently, the inverse of the maximum singular value of $M(j\omega)$ $\left(i.e., \bar{\sigma}^{-1}[M(j\omega)]\right)$ provides a measure (induced 2-norm) of the complex uncertainty allowable in the loop as a function of frequency ω. The minimum of the plot of $\bar{\sigma}^{-1}[M(j\omega)]$ provides a guarantee of the allowable uncertainty size for all ω.

One problem with the singular-value method is that it may model uncertainties in the system which are not physically meaningful, since it allows any input (to the Δ-block) to be coupled to any Δ-block output. The structured singular value μ provides an alternative measure of stability robustness which permits the analyst to remove unrealistic couplings. This is accomplished by prescribing a structure for the complex gain matrix $\Delta(s)$. To measure system stability robustness using μ analysis, the engineer first models system uncertainties as a complex gain matrix $\Delta(s)$ with a block diagonal structure, appropriately located in a flow path of a transfer-function-matrix block diagram. This structured-uncertainty matrix is composed of smaller complex uncertainty blocks (Δ_i-blocks) along its main diagonal. To permit μ analysis it is required simply that the resulting system, with Δ-block inserted, be capable of arrangement as shown in Fig. 4. Once this has been done, the structured singular value μ of $M(j\omega)$ can be found. (In actuality, μ can be determined exactly only if the number of Δ_i-blocks is three or fewer; for a larger number of blocks, upper and lower bounds must be employed [3].) $\mu^{-1}[M(j\omega)]$ provides a measure of the largest magnitude (induced 2-norm) that the Δ_i-blocks can all possess simultaneously without any combination of the Δ_i-blocks leading to system instability. The structured singular value $\mu[M(j\omega)]$, used with a structured uncertainty, is analogous to the maximum singular value $\overline{\sigma}[M(j\omega)]$ which was used in the unstructured case.

Example: Robust Stability Analysis for Actuator Uncertainties

To illustrate, consider a robust stability analysis for a microgravity isolation system with noncontacting magnetic actuators. Two types of actuators are under consideration/development: magnetic bearings and Lorentz actuators [7]. Each type is likely to experience gain variation as a function of payload position in the rattlespace. Position-dependent cross-coupling is likely as well, between each actuator's nominal line of force and the associated orthogonal axes. Finally, anomalous phase lags can be expected due to unmodeled physical phenomena (*e.g.*, eddy currents and hysteresis). Such actuator uncertainties can be modeled by use of a multiplicative input Δ-block, inserted into the nominal analysis model (closed-loop portion) as shown in Fig. 5. The Δ-block has a direct physical correspondence to phase-plus-gain uncertainties in the controller output(s) or actuator(s). The uncertainty can be either unstructured or structured. Assuming that each actuator has only one line of action. If a control input to one actuator affects only the output of that actuator, a structured uncertainty model will give more realistic (*i.e.*, less conservative) results. If there is appreciable cross-coupling from one actuator input to the output of another actuator (*e.g.*, via the cocking of the payload) then the unstructured form is more appropriate.

Consider the unstructured case first. In this case, Δ is a full complex matrix, with uncertainties between all of its inputs and outputs. Re-express the system in the form shown in Fig. 6, where M is the transfer-function matrix "seen" by Δ. For this example (*i.e.*, with a multiplicative input Δ-block),

$$M = G_2 G_1 (I - G_2 G_1)^{-1}. \tag{5}$$

W is a scalar weighting function, used to normalize the Δ-block so that its maximum singular value is less than one. In order to have a stable system, it is necessary that the product $\Delta(s)W(s)M(s)$ have a maximum singular value $\bar{\sigma}$ less than one. Accordingly, $\bar{\sigma}^{-1}[WM(j\omega)]$ gives the maximum allowable "size" of Δ as a function of ω. Specifically, if the maximum singular value of Δ is less than $\bar{\sigma}^{-1}[WM(j\omega)]$ there is no uncertainty represented by Δ that can destabilize a stable system. (*E.g.*, for a SISO system one could choose W to reflect the minimum acceptable phase- or gain variation from controller output to plant input, as a function of frequency [2]. With this normalization of Δ, if the plot of $\bar{\sigma}^{-1}[WM(j\omega)]$ is below unity for all ω the nominal system is guaranteed to have the required stability margin.)

Next consider a structured-uncertainty case, where Δ has a block-diagonal form. For a system with two control inputs there would be two Δ_i-blocks: $\Delta = \begin{bmatrix} \Delta_1 & O \\ O & \Delta_2 \end{bmatrix}$. Physically this means that uncertainties are considered to exist only "in-channel" between input(s) and output(s) of Δ. Again, re-express the system in the form shown in Fig. 6; M is defined as before. W in this case is a diagonal weighting *matrix*, with one scalar weighting function per channel of input to Δ. This weighting matrix is used to normalize *each* Δ_i-block so that its maximum singular value $\bar{\sigma}(\Delta_i)$ is less than one. In order to have a stable system it is necessary that the product $\Delta(s)W(s)M(s)$ have maximum singular value $\bar{\sigma}$ less than one, as before. Accordingly, $\mu^{-1}[WM(j\omega)]$ gives the maximum allowable "size" of the uncertainty blocks Δ_i, as a function of ω. Specifically, if the maximum singular value of each Δ_i-block is less than $\mu^{-1}[WM(j\omega)]$ there is no uncertainty represented by Δ that can destabilize a stable system. (*E.g.*, for a system with two control inputs one could choose W to reflect the minimum acceptable *in-channel* phase- or gain variation from each controller output to the associated plant input, as a function of frequency [2]. With this normalization of Δ, if the plot of $\overline{\mu}^{-1}[WM(j\omega)]$ lies below unity for all ω the nominal system is guaranteed to have the required stability margins.) Note that since the uncertainty block(s) Δ_i can represent complex gains in any direction in the gain plane, even those directions not corresponding to realistic variations, μ-analysis results are conservative in nature.

In the case of an unstructured Δ-block the plot of $\overline{\sigma}^{-1}(M)$ can be used to calculate MIMO gain margin (GM) and -phase margin (PM) ([3], pp. 47-56) at the plant input. In the structured-uncertainty case either $\overline{\sigma}^{-1}(M)$ or, less conservatively, $\mu^{-1}(M)$ can be used for this purpose. See the Appendix for details.

Other Stability-Robustness Checks for the Microgravity Isolation Problem

As shown above, multiplicative-input Δ-blocks are useful in finding guarantees of allowable variation for magnetic- or Lorentz actuators. Δ-blocks can be placed elsewhere in the system block diagram to provide other useful stability-robustness guarantees, in similar fashion. The controller inputs for the microgravity isolation problem typically come from relative-position sensors and accelerometers. Each sensor will filter its input in a manner only approximated by its transfer-function model. The analyst can determine guarantees of allowable sensor variations with the use of multiplicative-output Δ-blocks (Fig. 7a). As before, a MIMO GM and PM can be derived. In addition to the above variations, there will be higher system modes for which the payload- and umbilical models do not account. An "additive uncertainty" Δ-block can be placed in a forward direction around the plant (Fig. 7b) to find an measure of the modal uncertainties allowable. One can also use a feedback uncertainty-block (Fig. 7c) to model uncertainties in payload mass, or in umbilical stiffness or damping, as will be shown later.

Robust Performance

It is necessary also that the closed-loop system *performance* not degrade unacceptably if the isolation system model is inaccurate or if the system changes over time. For example, umbilical model uncertainties must not reduce microgravity isolation below acceptable levels. The level of performance robustness can be measured by using μ analysis, in a manner similar to that employed for stability-robustness analysis. By posing the closed-loop frequency-domain specifications in the form of a Δ-block with an appropriately selected weighting matrix (Δ_{pf} and W_{pf}, respectively), and at the same time using stability Δ-blocks (Δ_{st}) as described previously, the designer can determine how much complex uncertainty is allowable at various places in the closed-loop system, without exceeding the performance specifications or causing system instability ([3], pp. 68-73). Typically the non-zero elements of W_{pf} are the reciprocals of the appropriate closed-loop transfer functions. Again, the results are conservative.

Figs. 8a and b give examples of robust-performance analysis models which are useful for the microgravity isolation problem. The structured singular value plot for the multiplicative input (or -output) case can be used to determine a MIMO gain variation (GV) or -phase variation (PV). These are conservative measures of the gain- or phase variation allowable in each channel (entirely analogous to MIMO PM and GM) without violating performance specifications or losing stability guarantees. (See the Appendix for details.) These measures are found for a realistic 1-D microgravity isolation problem in reference [2].

Real Parametric Uncertainty Guarantees

The multiplicative-input-, multiplicative-output-, and additive-uncertainty blocks used in the above checks are inadequate by themselves to verify system stability- or performance robustness for the microgravity vibration isolation problem. In particular, these checks cannot verify system robustness to uncertainties in umbilical properties or in payload mass/inertia. These kinds of uncertainties are referred to as "real parametric uncertainties." Robustness in the face of real parametric uncertainties is of particular concern for the microgravity vibration isolation problem, since umbilicals are quite difficult to model accurately. Some possible approaches to this problem are surveyed below, along with a new approach using complex feedback uncertainty.

Real-μ

One possible approach to determining real-parametric-uncertainty guarantees is the use of real-μ (μ_R). To use μ_R the problem is first expressed in the form of Fig. 6, where the diagonal elements Δ_i of the Δ-block are restricted to being real and uncorrelated. (The initial procedure for producing this form is generally somewhat involved, but straightforward. See [8].) The interpretation of μ_R is analogous to that of complex-μ, with the exception that now only real gains are considered. Unfortunately, exact calculations of μ_R are very computationally intensive; accordingly, its practical use is severely limited, to about eight or nine parameters [9]. For some problems, though, this is an acceptable limitation. If the 3-D microgravity problem has umbilicals that can be modeled jointly by one 3-D spring-and-damper system, and a payload that is isolated by one 3-D magnetic actuator, the real-μ problem can have one mass, three uncorrelated inertias, three uncorrelated stiffnesses, and three uncorrelated dampings. If the uncertainties in damping can be considered of negligible effect (as indicated by certain studies of the 1-D problem [2]), then real-μ could be a practical analysis option. Note, however, that real-μ cannot be used for performance-robustness analysis, since only real uncertainties can be represented in the corresponding Δ-block.

Mixed-μ

A second approach to obtaining real-parametric-uncertainty guarantees is to treat the problem using mixed-μ (μ_κ), where both real and complex uncertainties (including repeated blocks) are allowed in the Δ-block substructure [10]. In general, μ_κ is more conservative than μ_R, but the inclusion of complex gains permits performance-robustness checks while still considering real parametric uncertainties. Again, the starting point is shown in Fig. 6. As with real-μ, exact mixed-μ calculation appears to be fundamentally limited to very restricted cases. However, there have been promising advances in finding useful upper and lower bounds on μ_κ for many engineering problems of practical size (with as many as 100 real parameters) [10].

Complex-μ

A third approach would be to set up the problem as with real-μ, but to compute complex-μ (μ) instead. This method can be used for both stability- and performance-robustness checks. However, μ is more conservative than μ_κ or μ_R, and there is still no exact computational method for more than three Δ_i-blocks. Again, one must generally settle for upper and lower bounds.

Complex-μ with Feedback Uncertainty

Alternatively, complex-μ can be used in a fundamentally different way for conducting real-parametric-uncertainty analysis. The previous three methods all require the use of separate Δ_i-blocks for each parameter under consideration, so that the respective parametric uncertainties are considered in mutual isolation. For the 1-D microgravity isolation problem, this means that δk, δc, and δm must each have its own Δ_i-block in the overall uncertainty-block structure. However, for this problem the uncontrolled plant has a characteristic equation (*viz.*, $ms^2 + cs + k$) that permits exploitation for real-parametric-uncertainty analysis. If the plant is rearranged appropriately for the insertion of a complex feedback Δ-block, it is possible to use the particular structure of the associated complex-μ to determine combinations of payload mass, umbilical stiffness, and umbilical damping for which robust stability can be assured. (As with the preceding methods, the results are conservative.)

Consider the block diagram shown in Fig. 9a, where δm, δc, and δk represent *real* variations in payload mass, umbilical damping, and umbilical stiffness, respectively. This block diagram reduces to the equivalent block diagram shown in Fig. 9b. If a *complex* Δ-block, "$\Delta_{F/B}$," is placed in a negative feedback path around $\dfrac{1}{ms^2 + cs + k}$ (*i.e.*, in place of the feedback transfer function in Fig. 9a), then *real* parametric uncertainty bounds on k, c, and m can be obtained from the structured singular value plot of the system "seen" by $\Delta_{F/B}$. The method is as follows, for a one-dimensional (1-D) isolation problem.

First, arrange the closed-loop system such that it contains a transfer function block having the structure $\frac{1}{ms^2 + cs + k}$. For a 1-D microgravity isolation system such a block diagram is shown in Fig. 10. To evaluate the stability robustness of this system, one can look at a rearranged (and abbreviated) block diagram, shown in Fig. 11, where a feedback uncertainty block $\Delta_{F/B}$ has been inserted into the analysis model. This feedback stability robustness model can be reduced readily to the form of Fig. 4, with the plant "seen" by $\Delta_{F/B}$ designated $M_{F/B}$. For a given value of ω (say, ω_1) $\Delta_{F/B}(\omega_1)$ can be conceptualized as a ball in the complex gain plane, of radius $r_{\Delta}(\omega_1)$, with radial unit direction vector (variable) $\hat{g}$. The value $r_k(\omega_1) = \mu^{-1}[M_{F/B}(j\omega)]$ gives the maximum allowable size (2-norm) of complex uncertainty $\Delta_{F/B}(\omega_1)$. In particular, if $r_{\Delta}(\omega_1)$ is smaller than $r_k(\omega_1)$ there exists no complex unit direction vector $\hat{g}$ at which an uncertainty (gain) represented in $\Delta_{F/B}(\omega_1)$ will drive a stable system unstable. However, a $\Delta_{F/B}(\omega_1)$ of size $r_{\Delta}(\omega_1) = r_k(\omega_1)$ will drive a system pole onto the $j\omega$-axis for some complex unit direction $\hat{g} = \hat{g}_o$. The result will be system instability, for the complex uncertainty $r_k(\omega_1)\hat{g}_o$. (This complex uncertainty might not, of course, actually correspond to a physically possible variation. Accordingly, the term "allowable" may be true only in a conservative sense.)

Note that the above discussion is for a *particular* (arbitrary) frequency, ω_1. When the plotted magnitude of $\Delta_{F/B}(\omega)$ remains below the curve $r_k(\omega)$ for *all* values of ω, this means that there is no complex uncertainty contained in the uncertainty ball that can drive the system unstable at *any* frequency ω.

Let Δk represent a complex ball of potential variations in stiffness k. Then the plot $r_k(\omega)$ displays the "allowable" (in the sense just described) size of Δk, as a function of ω, given no uncertainty in c or m. In other words, at any frequency ω, $r_k(\omega)$ indicates how large Δk can be without placing a system pole on the $j\omega$-axis at $s = j\omega$. (Since only real variations in stiffness are physically possible, the plot of $r_k(\omega)$ provides a conservative guarantee of permissible stiffness variations as a function of frequency ω. In this sense, it provides guarantees but not limits on permissible stiffness variations.) If one defines r_c, Δc, r_m, and Δm analogously, the following can be obtained:

$$r_k(\omega) = \mu^{-1}\left(M_{F/B}\Big|_{s=j\omega} \right) \tag{6a}$$

$$r_c(\omega) = \mu^{-1}\left(sM_{F/B}\Big|_{s=j\omega} \right) = \frac{r_k(\omega)}{\omega} \tag{6b}$$

$$r_m(\omega) = \mu^{-1}\left(s^2M_{F/B}\Big|_{s=j\omega} \right) = \frac{r_c(\omega)}{\omega} = \frac{r_k(\omega)}{\omega^2} \tag{6c}$$

The minimum of each curve (call these minima ρ_k, ρ_c and ρ_m, respectively) tightly bounds the amount of complex variation "allowable" (again, in the conservative sense described above) in Δk, Δc, and Δm, respectively, without any other feedback variations. If, for example, the size of Δk equals the minimum ρ_k, instability will occur at some frequency ω_k for the complex variation Δk of that magnitude, in some complex direction (call the unit direction vector $\hat{g}_k$). Analogous situations exist in the cases of Δc and Δm. ρ_k, ρ_c and ρ_m thus provide the designer with conservative guarantees of "allowable" real parametric plant uncertainties δk, δc, and δm, *taken one at a time*.

It is possible also to use the structured singular value information to obtain guarantees on *combinations* of real parametric uncertainties. Let $\mathcal{A}_r$ be defined as an "acceptable region" in the real parameter space made up of points with coordinates (δk, δc, δm). The origin $(0,0,0)$ corresponds to a system with nominal plant parameters $k = k_{nom}$, $c = c_{nom}$, and $m = m_{nom}$. (Refer to Fig. 12a.) Let Γ_r be the boundary between $\mathcal{A}_r$ and the "unacceptable region," $\mathcal{U}_r$. Used in this way, "acceptable" and "unacceptable" refer to system behavior, in terms of stability (for the present consideration) or performance. For a given frequency ω, there may be some combinations $(\delta k,\ \delta c, \delta m)$ for which the control results in an acceptable (*e.g.*, stable) behavior; such points would be in $\mathcal{A}_r$. Other values $(\delta k,\ \delta c, \delta m)$ which yield an unacceptable behavior at that frequency would be in $\mathcal{U}_r$. The boundary Γ_r is found, in theory, by varying s from $j0$ to $j\infty$, and plotting each point (δk, δc, δm) that leads to a closed-loop system pole on the $j\omega$-axis. Only purely imaginary values of s are used.

The designer would like to bound δk, δc, and δm such that he remains in $\mathcal{A}_r$. One possible approach would be to use a "brute-force" (*i.e.*, point-by-point) method. Such an approach, while having the advantage of remaining entirely inside a *real* parameter space, can only make guarantees about the specific sample *points* considered. μ analysis, in constrast, considers all values in a *region* of a *complex* gain plane, so that the Δ-block is a "ball" that leads to conservative guarantees for complex uncertainties less than some "size." This information, although conservative, is still of great value; and since it is much easier to obtain, it has been used in this study. These *complex* uncertainties can be used to obtain guarantees on combinations of *real* parametric uncertainties, as follows.

Consider Fig. 12b. The region $\Omega(\omega_1)$ consists of all points in complex gain space corresponding to values of uncertainty gain $\underline{g}$ (*i.e.*, of the structured singular value of $\Delta_{F/B}$) which place a system pole on the $j\omega$-axis at frequency ω_1. ($\underline{g}$ is underlined to indicate that it is a vector quantity with real and imaginary components.) The "acceptable region" $\mathcal{A}(\omega_1)$ is a ball in the gain space, centered at $\underline{g}=\underline{0}$, with radius $r_k(\omega_1) = \mu^{-1}[M_{F/B}(j\omega_1)]$, such that it just touches the nearest point of Ω. A separate figure would exist, theoretically, for each value of ω.

An uncertainty block $\Delta_{F/B}$ whose magnitude (*i.e.*, radius in the gain-plane) is less than $r_k(\omega_1)$ is guaranteed not to represent any complex gains which could place a pole on the $j\omega$-axis at ω_1. The maximum allowable size of $\Delta_{F/B}$ (*i.e.*, of $r_k(\omega)$) varies with frequency; define ρ_k as the size of the largest uncertainty ball $\Delta_{F/B}$ (or alternatively, Δk) that will not lead to instability at *any* frequency. Let ω_k be the limiting frequency. ρ_c and ρ_m (with ω_c and ω_m, respectively) provide analogous conceptual pictures. Referring to Fig. 12b again, let $\underline{r}_k(\omega_1)$ represent the vector of $\Delta_{F/B}$ with complex gain $\underline{g} = \underline{g}_k$, componentiated in terms of its real and imaginary parts. $\underline{g}_k(\omega_1)$ is the limiting value of $\underline{g}(\omega_1)$ at frequency ω_1. Let $\hat{\underline{\sigma}}_g$ and $\hat{\underline{\nu}}_g$ be unit vectors in the real and imaginary $\underline{g}$-plane directions, respectively; let $\hat{\underline{g}}_k(\omega_1)$ be the unit vector in the direction of $\underline{r}_k(\omega_1)$; and let $\beta_1(\omega_1)$ and $B_2(\omega_1)$ be the scalar components of $\hat{\underline{g}}_k(\omega_1)$, such that

$$\underline{r}_k(\omega_1) = r_k(\omega_1)[\beta_1(\omega_1)\hat{\underline{\sigma}}_g + \beta_2(\omega_1)\hat{\underline{\nu}}_g], \quad \underline{r}_k(\omega_1) = r_k(\omega_1)\hat{\underline{g}}_k(\omega_1), \qquad \text{(7a,b)}$$

and $$\beta_1^2(\omega_1) + \beta_2^2(\omega_1) = 1 \qquad \text{(7c)}$$

Analogous vectors and equations exist for each frequency ω.

If, for all $\omega \leq 1$, $(\delta k - \omega^2 \delta m)^2 < \beta_1^2(\omega) r_k^2(\omega)$ and $(\omega \delta c)^2 < \beta_2^2(\omega) r_k^2(\omega)$ (8a,b)

where δk, δc, and δm are real variations in k, c, and m from the nominal, then one can be assured that no instability will occur for those values of ω. Since ρ_k is the $\min_{\omega}[r_k(\omega)]$, one can replace $r_k(\omega)$ with ρ_k in (8a,b) and the assertion will still hold. Finally, since the limiting case of β_1 and β_2 (*i.e.*, corresponding to ρ_k) is at a point of tangency to $\Omega(\omega_k)$, any other values γ_1 and γ_2 may be substituted for β_1 and β_2, provided that $\gamma_1^2 + \gamma_2^2 \leq 1$, without exceeding the boundaries of $\Omega(\omega_k)$. Then the following assertion will hold (Assertion #1):

If, for all $\omega \leq 1$, $(\delta k - \omega^2 \delta m)^2 < \gamma_1^2 \rho_k^2$ and $(\omega \delta c)^2 < \gamma_2^2 \rho_k^2$, (9a,b)

where $\gamma_1^2 + \gamma_2^2 \leq 1$, (9c)

then no instability will occur for those values of ω.

Defining $\beta_3(\omega)$ and $\beta_4(\omega)$ in an analogous way to $\beta_1(\omega)$ and $\beta_2(\omega)$, where $s^2 M_{F/B}$ now replaces M in Fig. 4 (refer also to Eq. 6c), one can also arrive at the following:

If, for all $\omega \geq 1$, $\left(\delta m - \frac{\delta k}{\omega^2}\right)^2 < \beta_3^2(\omega)\rho_m^2$ and $\left(\delta c/\omega\right)^2 < \beta_4^2(\omega)\rho_m^2$, (10a,b)

then no instability will occur for those values of ω.

Finally, one can replace $\beta_3(\omega)$ and $\beta_4(\omega)$ with variables γ_3 and γ_4, as before, provided $\gamma_3^2 + \gamma_4^2 \leq 1$. This results in the following, Assertion # 2:

If, for all $\omega \geq 1$, $\left(\delta m - \frac{\delta k}{\omega^2}\right)^2 < \gamma_3^2 \rho_m^2$ and $\left(\delta c/\omega\right)^2 < \gamma_4^2 \rho_m^2$, (11a,b)

where $\gamma_3^2 + \gamma_4^2 \leq 1$, then no instability will occur for those values of ω. (11c)

From Assertions #1 and #2 the following condition can be obtained:

Given any γ_1, γ_2, γ_3, and γ_4 satisfying $\gamma_1^2 + \gamma_2^2 \le 1$ and $\gamma_3^2 + \gamma_4^2 \le 1$, and with ρ, ρ_c, and ρ_m as previously defined, the following stability condition holds:

$$\left\{\begin{array}{l} |\delta k| < \min\left\{\begin{array}{c} \rho_k \\ \gamma_1\rho_k - |\delta m| \\ \gamma_3\rho_k - |\delta m| \end{array}\right\} \\ \text{and} \quad |\delta m| < \rho_m \\ \text{and} \quad |\delta c| < \min\left\{\begin{array}{c} \rho_c \\ \gamma_2\rho_k \\ \gamma_4\rho_m \end{array}\right\} \end{array}\right\} \Rightarrow \text{system stability (Assertion \#3)}$$

By using Assertion #3, one can obtain guarantees of real variations in umbilical stiffness, umbilical damping, and payload mass that can occur simultaneously without violating stability specifications. Upon obtaining ρ_k, ρ_c, and ρ_m from the appropriate structured-singular-value plots, the analyst chooses γ_2 and γ_4 based on the percent error expected in c. Next, he chooses an expected percent error in m (such that $|\delta m| < \rho_m$). Then γ_1 and γ_3 are found from the equations $\gamma_1^2 + \gamma_2^2 = 1$ and $\gamma_3^2 + \gamma_4^2 = 1$; and finally guarantees are determined on allowable δk. This provides the analyst with a guarantee on the allowable maximum magnitude of stiffness variation δk, as a function of the assumed maximum magnitudes of simultaneous variations in umbilical damping δc and payload mass δm. For any combination of real parametric variations remaining within this real parameter space, the analyst can guarantee that the system will be stable.

This analysis procedure depends on the ability to arrange the microgravity isolation-system model and complex-feedback-uncertainty block(s) in the appropriate form. Application to the 1-D case is found in reference [2]. The method was found to give useful results for stability-robustness analysis, but the performance-robustness results were excessively conservative. For more complicated geometries the application will be more difficult, and may not always be possible. Nonetheless, the use of feedback uncertainty Δ-block(s) can provide the engineer with at least a relative measure of closed-loop-system robustness to umbilical- and payload parametric variations.

Concluding Remarks

Analysis is a vital part of controller design for a microgravity vibration-isolation problem. Due to the many competing requirements for a microgravity isolation system, the design of an effective controller involves a studied, iterative process of synthesis and analysis. Each synthesized controller must be subjected to a series of analytical checks, to verify that it will perform satisfactorily under even the most pessimistic combination of possible model inaccuracies.

Most of the types of uncertainty that are of concern with a microgravity isolation system can be modeled by complex uncertainty-blocks, appropriately placed in the nominal system's transfer-function block diagram. This includes uncertainties in amplifier, actuator, and sensor models, as well as unmodeled higher modes of the system. The powerful methods of complex-μ analysis permit the analyst to obtain guarantees on the amount of uncertainty of each type that can be tolerated without compromising stability or violating performance constraints.

Guarantees on allowable variations in real system parameters (such as umbilical stiffness and damping, and payload mass/inertia) may be found by similar, if more sophisticated, techniques. These techniques generally require mutual isolation of selected parameter uncertainties in the uncertainty-block structure. The analyst then obtains the desired guarantees by using either real-, mixed-, or complex-μ methods, in increasing order of conservatism. Alternatively, the form of the microgravity isolation problem permits an approach involving the use of complex-μ with a feedback uncertainty-block having a much simplified structure. The particular relationships between the parametric uncertainties, which are represented intermingled in the feedback uncertainty-block, have been exploited to determine allowable combinations of real-parameter variations.

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References

[1] NELSON, EMILY S., "An Examination of Anticipated g-Jitter on Space Station and Its Effects on Materials Processes," *NASA TM-103775*, April 1991.

[2] HAMPTON, R. D., "Controller Design for Microgravity Vibration Isolation Systems," Ph.D. dissertation, University of Virginia, Charlottesville, Virginia, 1993.

[3] DAILEY, R. LANE. "Lecture Notes for the Workshop on H_∞ and μ Methods for Robust Control," Seattle, Washington: The Boeing Company, presented at the 1990 American Control Conference, San Diego, California, May 21-22, 1990.

[4] HAMPTON, R. D.; KNOSPE, C. R.; ALLAIRE, P. E.; and GRODSINSKY, C. M., "Microgravity Isolation System Design: A Modern Control Synthesis Framework," *NASA TM-106805*, December, 1994.

[5] MOORE, B. C. "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," *IEEE Transactions on Automatic Control*, Vol. AC-26, 1981, pp. 17-32.

[6] SAFONOV, M. G.; LAUB, A. J.; and HARTMANN, G. L. "Feedback Properties of Multivariable Systems: The Role and Use of the Return Difference Matrix," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 1, February 1981, pp. 47-65.

[7] ALLAN, A. P. and KNOSPE, C. R. "A Six Degree-of-Freedom Magnetic Bearing for Microgravity Vibration Isolation," *Proceedings of the International Symposium on Magnetic Suspension Technology*, Hampton, Virginia, August 19-23, 1991, sponsored by NASA Langley Research Center.

[8] MORTON, BLAISE G. and MCAFOOS, ROBERT M. "A Mu-test for Robustness Analysis of a Real-Parameter Variation Problem," *Proceedings of the American Control Conference*, pp. 135-138, Boston, Massachusetts, June 19-21, 1985.

[9] DAILEY, R. LANE. "A New Algorithm for the Real Structured Singular Value," *Proceedings of the American Control Conference*, pp. 3036-3040, San Diego, California, May 23-25, 1990.

[10] YOUNG, PETER M.; NEWLIN, MATTHEW P.; and DOYLE, JOHN C. "Practical Computation of the Mixed μ Problem," *Proceedings of the American Control Conference*, pp. 2190-2194, Chicago, Illinois, June 24-26, 1992.

Appendix

The familiar phase margin (PM) and gain margin (GM) can be generalized to apply to multiplicative input (or output) uncertainties for MIMO systems. Dailey's discussion [3] is adapted below. Let the ith control-input channel have a multiplicative phase rotation θ_i, (*i.e.*, $1 + \Delta_i = e^{j\theta_i}$), and let the various phase rotations be independent of each other. The input MIMO PM is defined as the largest real (unique) interval $Int = [-\theta, \theta]$ such that for all simultaneous independent phase rotations $\theta_i \in Int$ $(i=1, \ldots, n)$ the system remains stable. The output MIMO PM is defined similarly. An input MIMO GM is defined as a real (non-unique) interval $Int = [G_L, G_U]$ such that for all simultaneous independent gain variations G_i satisfying $1 + G_i \in Int$ $(i = 1, \ldots, n)$ the system remains stable. The output MIMO GM is similarly defined.

Let S_1 and T_1 be the sensitivity and complementary sensitivity transfer matrices, respectively, at the plant input. Then, in terms of the complementary sensitivity function T_1, a guaranteed lower bound for the MIMO PM is given by

$$\text{MIMO PM} \supseteq [-\theta, +\theta] \tag{A-1a}$$

$$\text{where } \theta = 2\sin^{-1}\left(\frac{r_{min}}{2}\right) \tag{A-1b}$$

$$\text{for } r_{min} = \inf_{\omega \in R} \bar{\sigma}^{-1}[T_1(j\omega)] \tag{A-1c}$$

A valid MIMO GM is given by

$$\text{MIMO GM} = [1 - r_{min}, 1 + r_{min}] \tag{A-2}$$

In terms of the sensitivity function S_1,

$$\text{MIMO PM} \supseteq [-\theta, +\theta] \tag{A-3a}$$

$$\text{where } \theta = 2\sin^{-1}\left(\frac{r_{min}}{2}\right) \tag{A-3b}$$

$$\text{for } r_{min} = \inf_{\omega \in R} \bar{\sigma}^{-1}[S_1(j\omega)], \tag{A-3c}$$

$$\text{and } \text{MIMO GM} = \left[\frac{1}{1 + r_{min}}, \frac{1}{1 - r_{min}}\right]. \tag{A-4}$$

If instead of $\bar{\sigma}$ the structured singular value μ is used in Eqs. (A-1c) and (A-3c) above, the lower bounds on the stability margins will be improved. For the analogous output stability margin guarantees, one merely substitutes the output sensitivity and complementary sensitivity transfer matrices S_2 and T_2, respectively, into the appropriate equations given above.

Application to cases where performance specifications are included as Δ-blocks can be made readily using structured singular values. In these cases, however, the terms "MIMO phase variation (PV)" and "MIMO gain variation (GV)" are more appropriate, for obvious reasons.

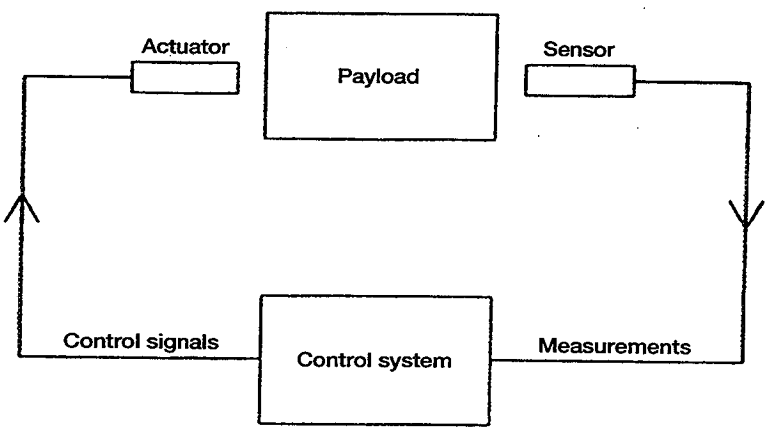

Figure 1.—Vibration isolation system.

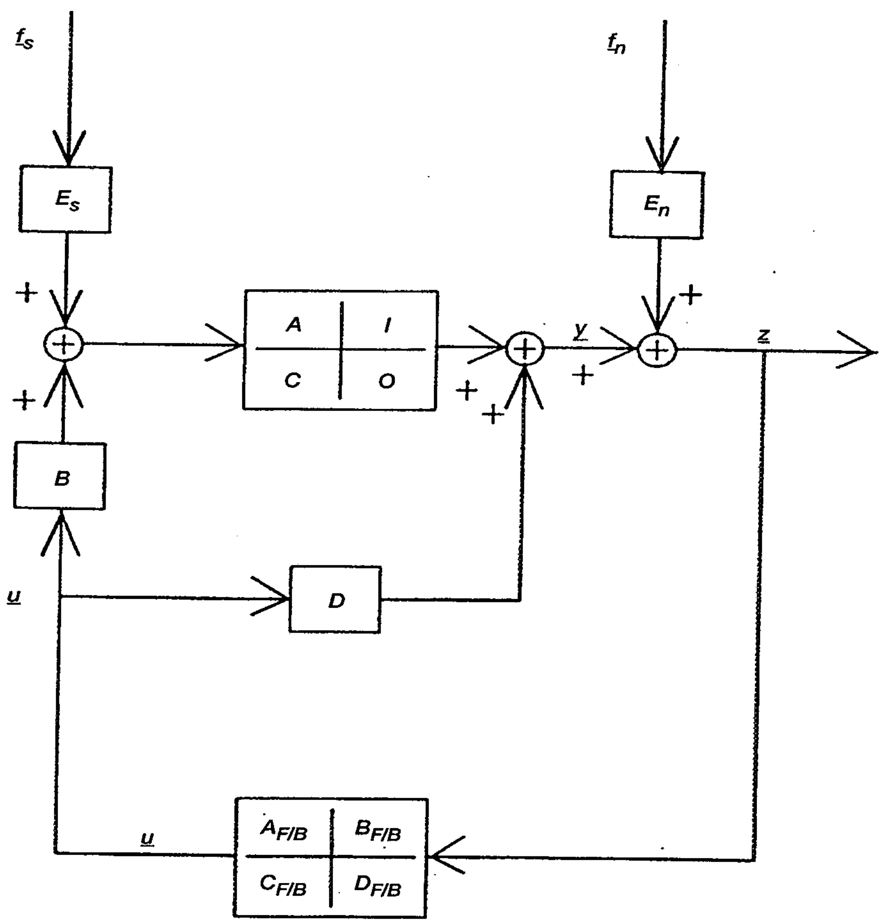

Figure 2.—Nominal analysis model.

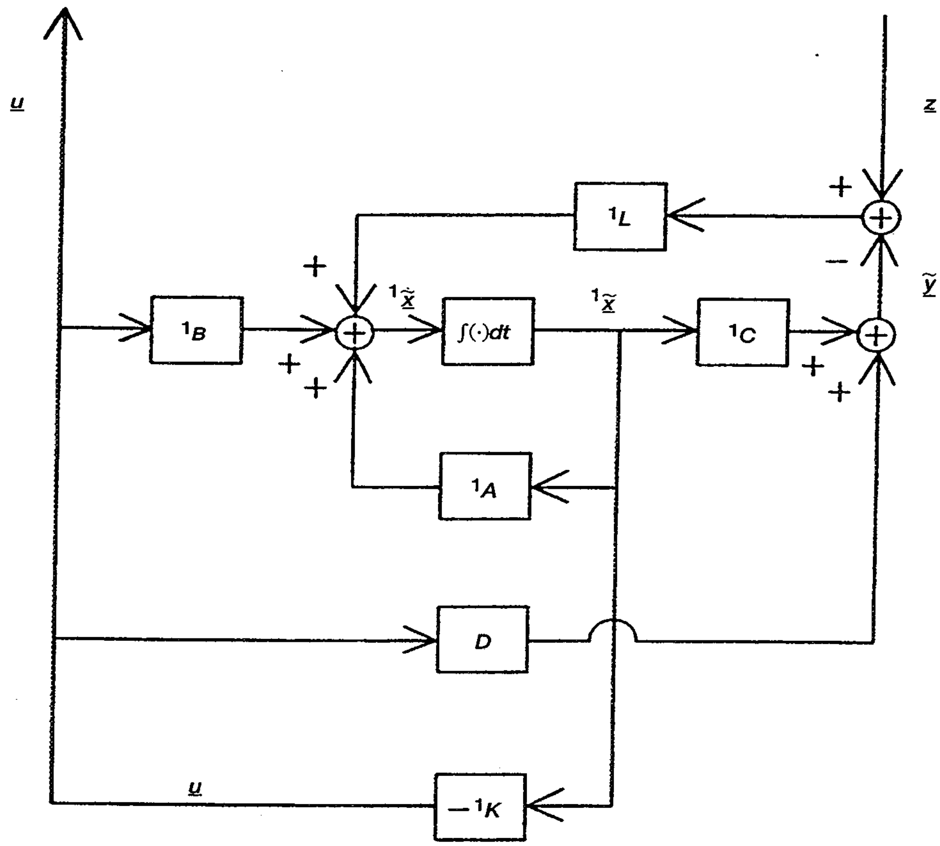

Figure 3a.—Full observer-synthesis model.

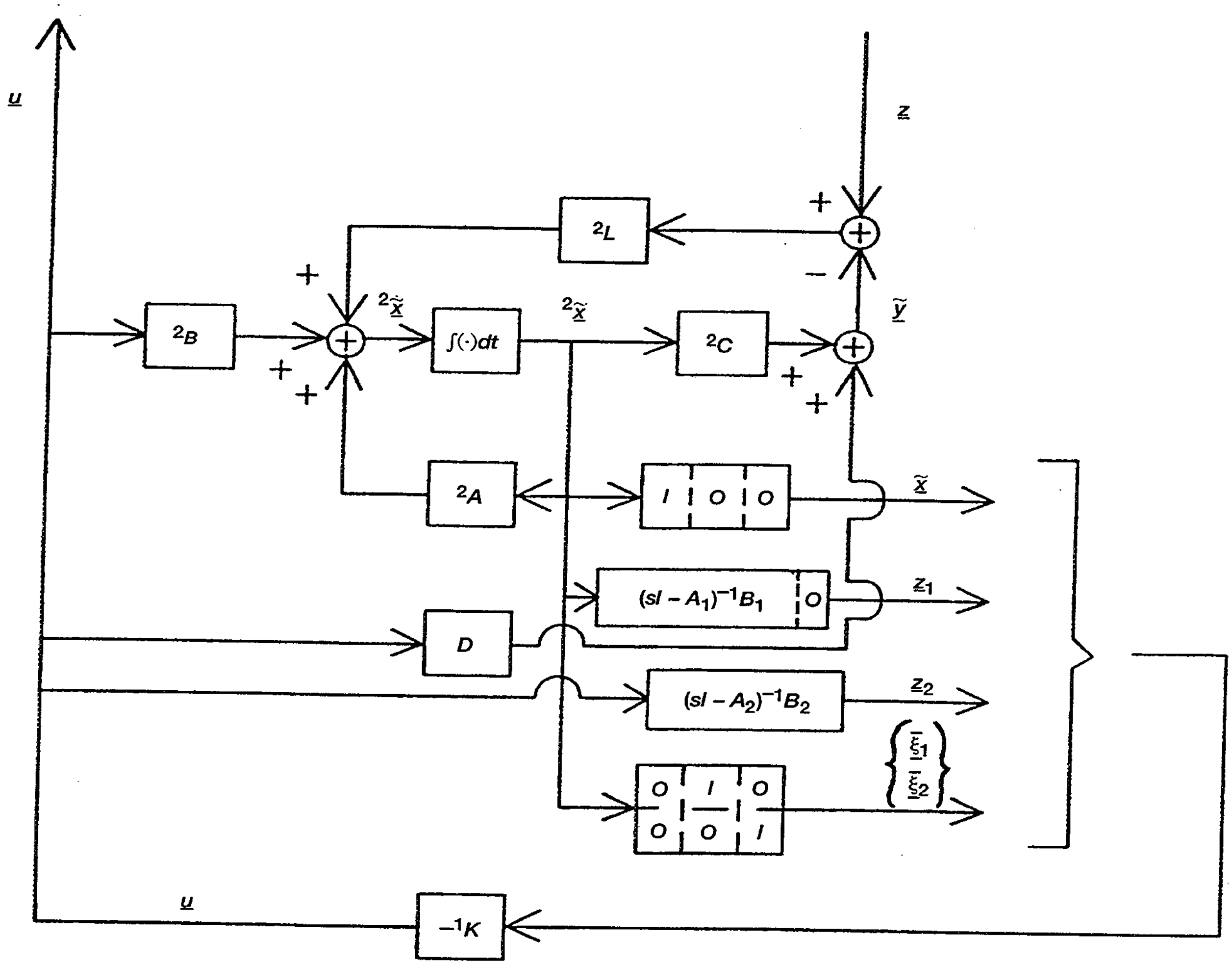

Figure 3b.—Reduced observer synthesis model.

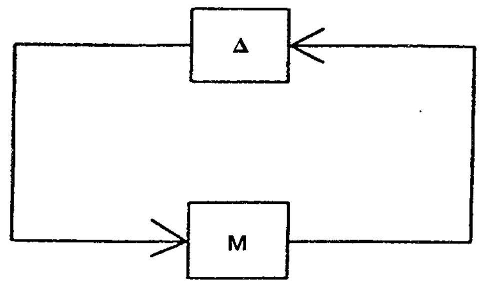

Figure 4.—Basic uncertainty analysis model.

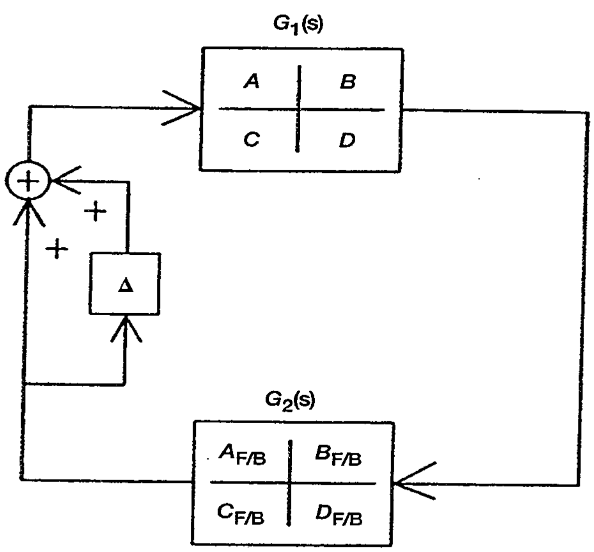

Figure 5.—Robust stability analysis model: multiplicative input uncertainty.

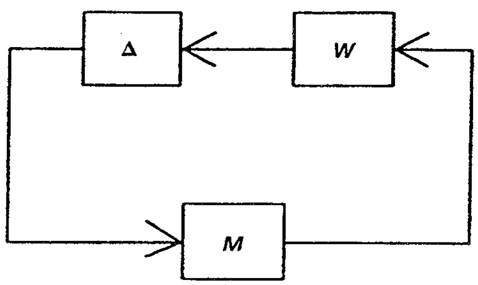

Figure 6.—Weighted μ-analysis model.

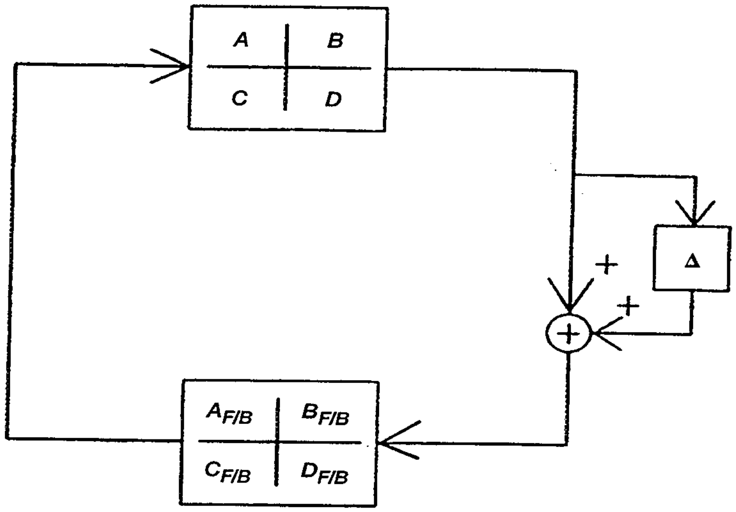

Figure 7a.—Robust stability analysis model: multiplicative output uncertainty.

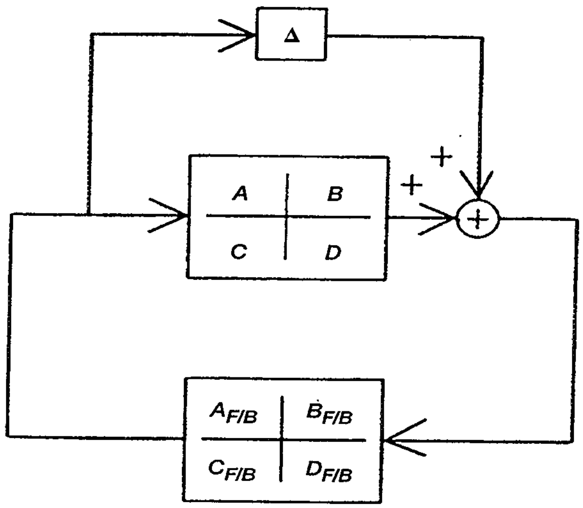

Figure 7b.—Robust stability analysis model: additive plant uncertainty.

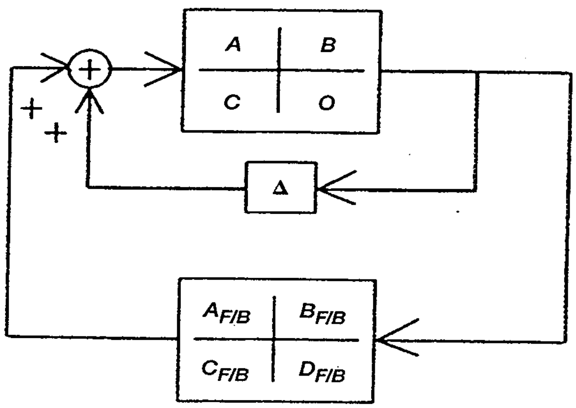

Figure 7c.—Robust stability analysis model: feedback uncertainty.

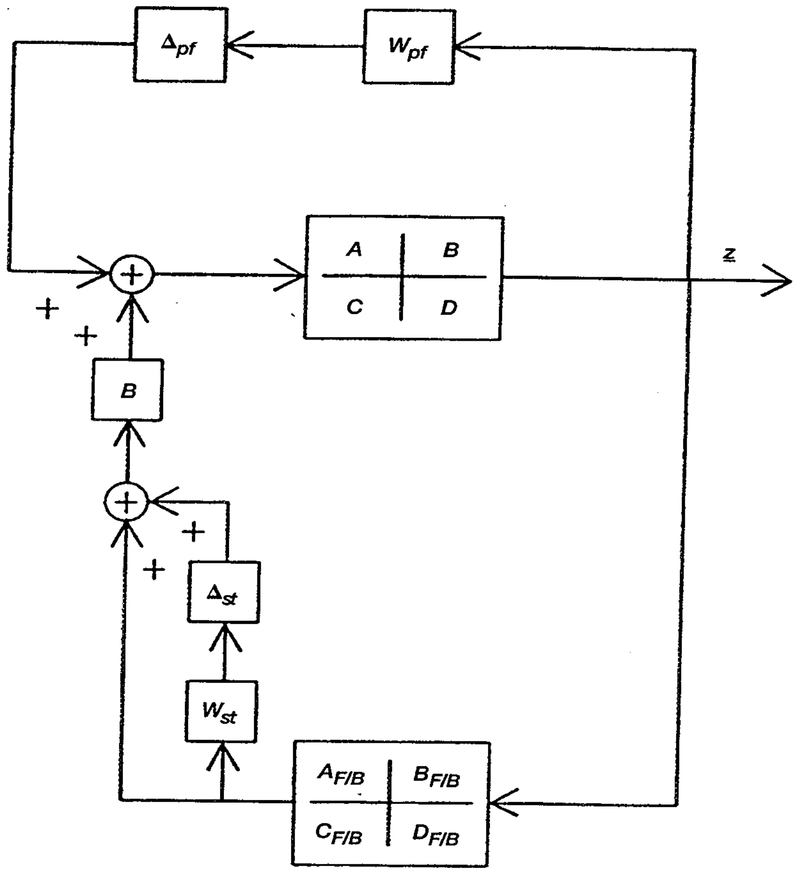

Figure 8a.—Robust performance analysis model: multiplicative input uncertainty.

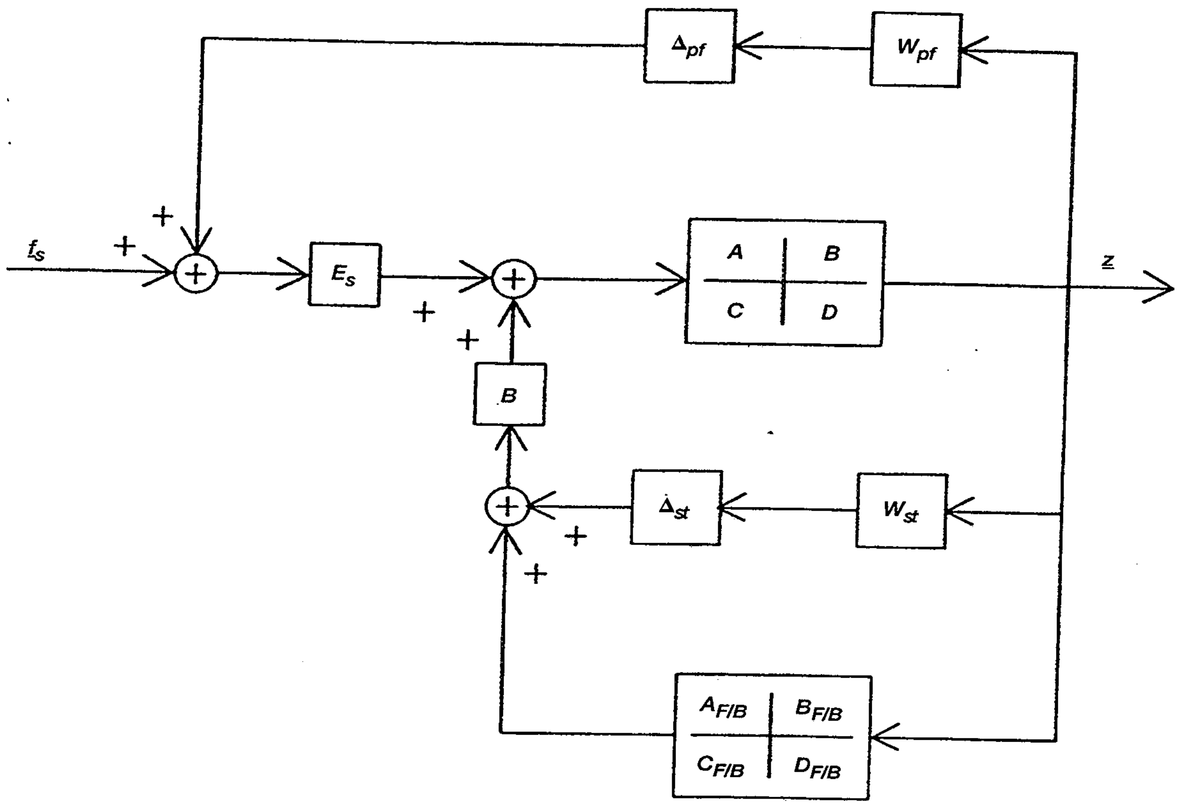

Figure 8b.—Robust performance analysis model: feedback uncertainty with input disturbance model.

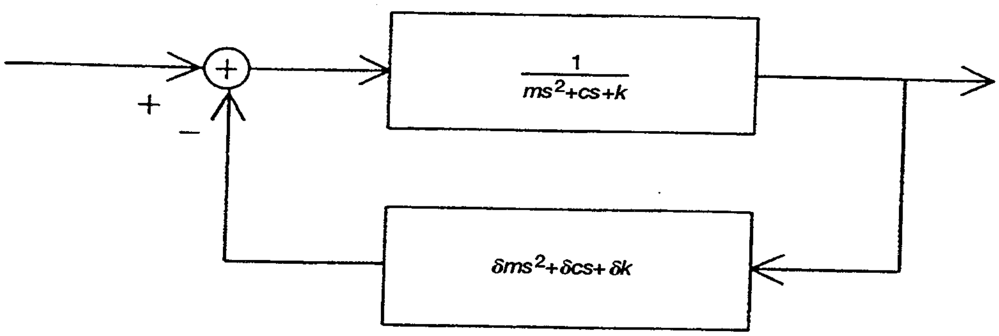

Figure 9a.—Feedback variation block diagram.

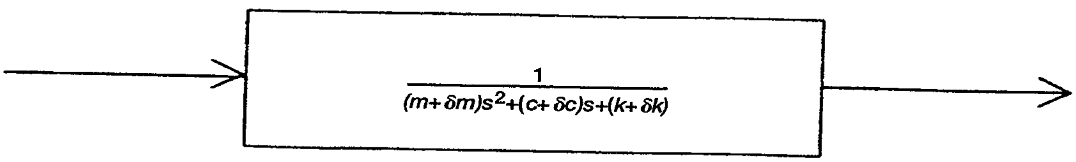

Figure 9b.—Reduced feedback variation block diagram.

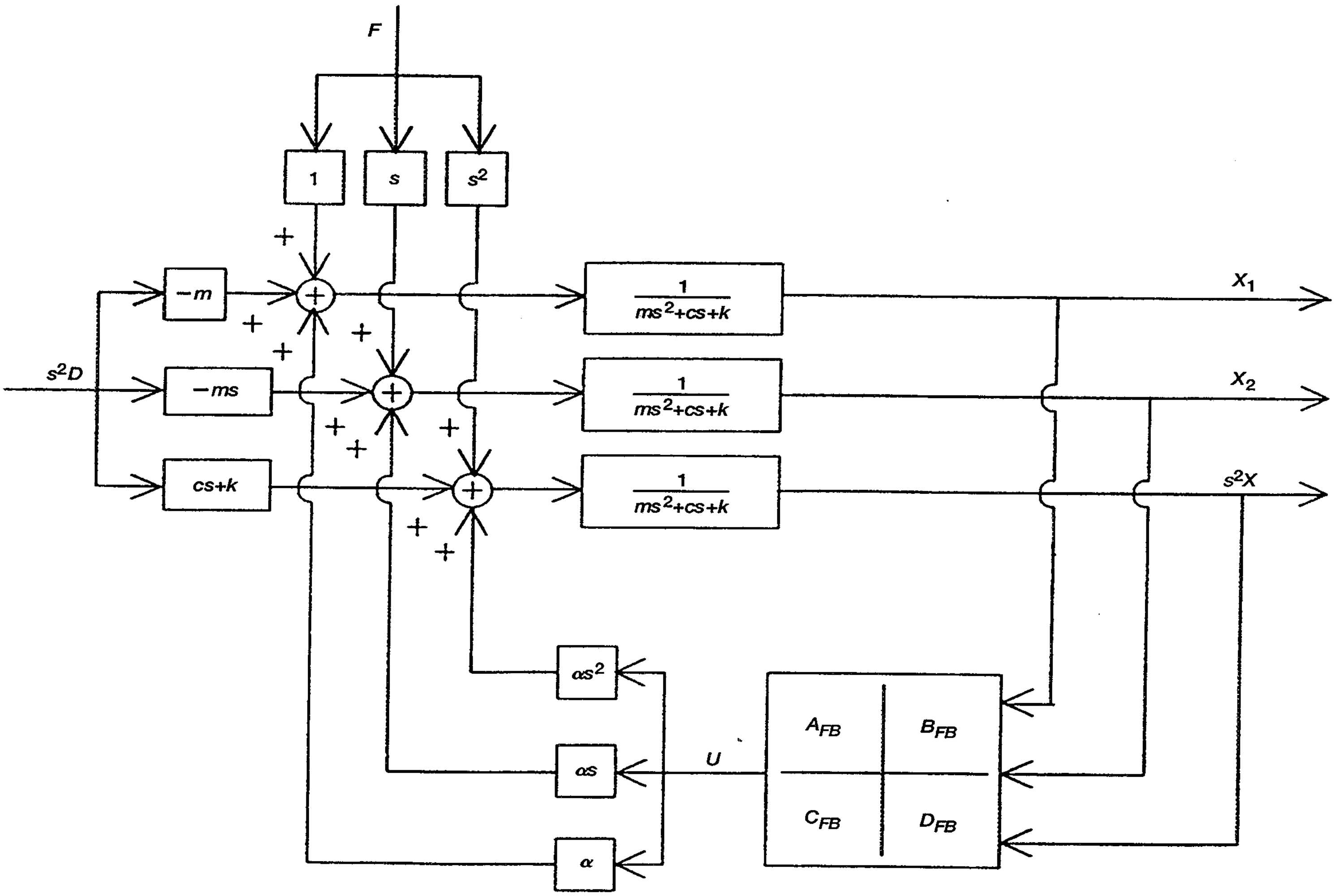

Figure 10.—Closed-loop system block diagram, frequency domain.

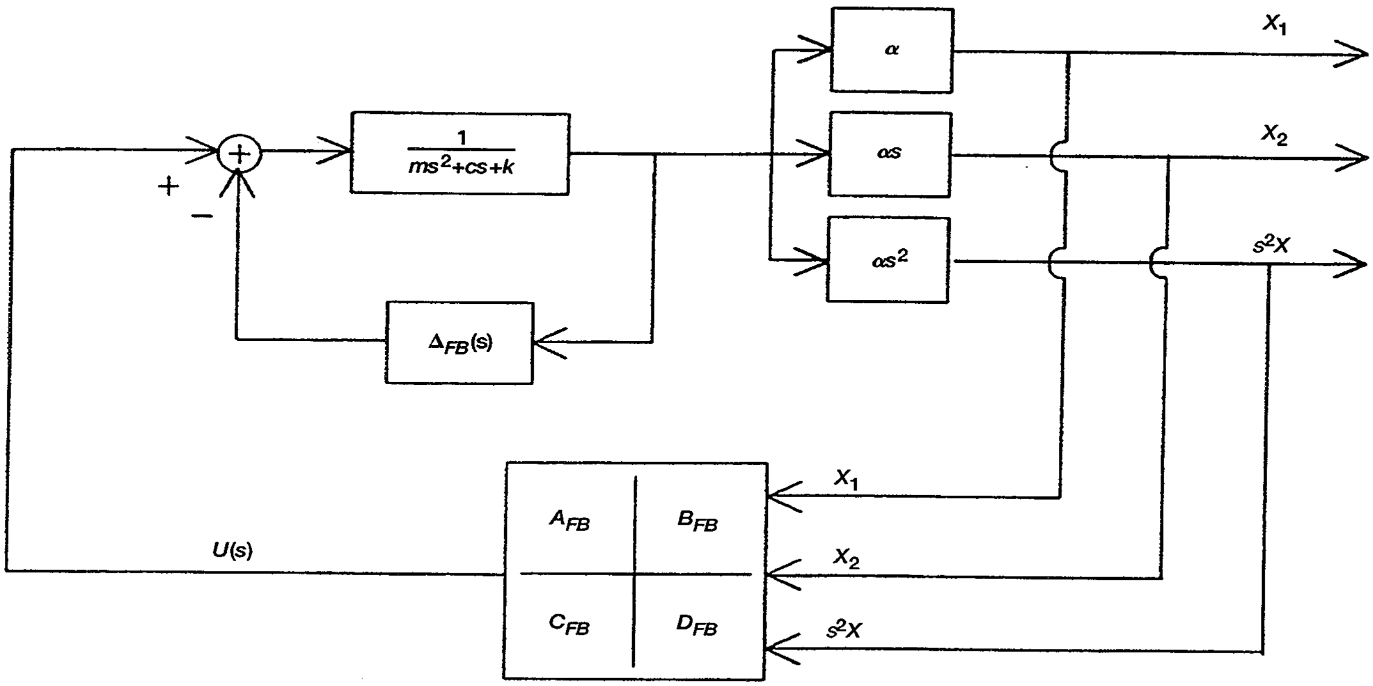

Figure 11.—Block diagram for feedback stability robustness analysis.

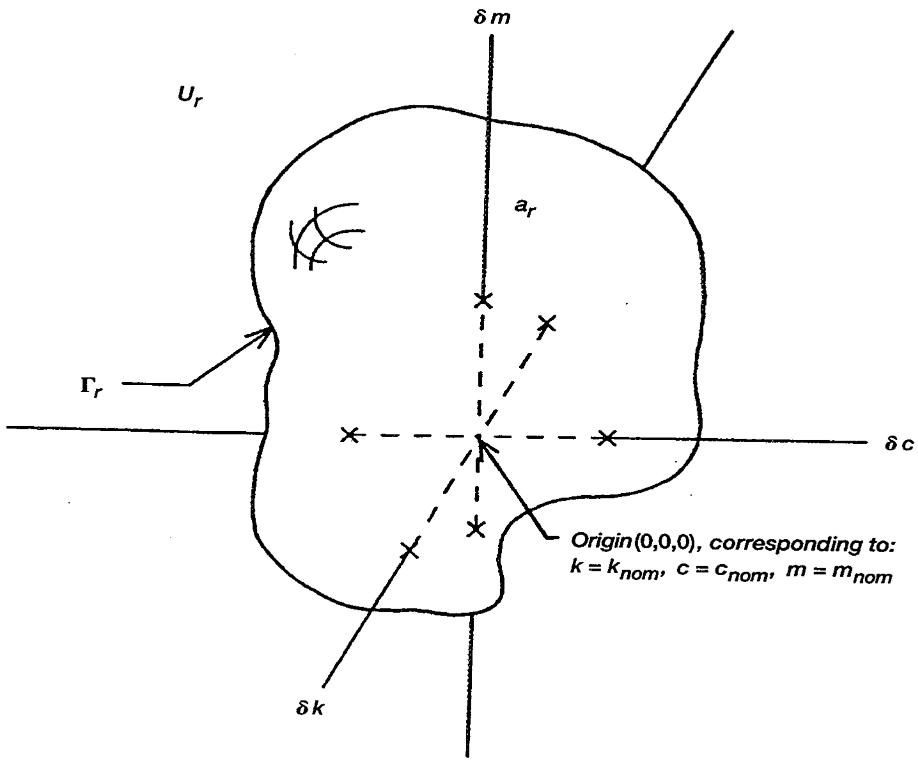

Figure 12a.—Real parametric uncertainty acceptable region.

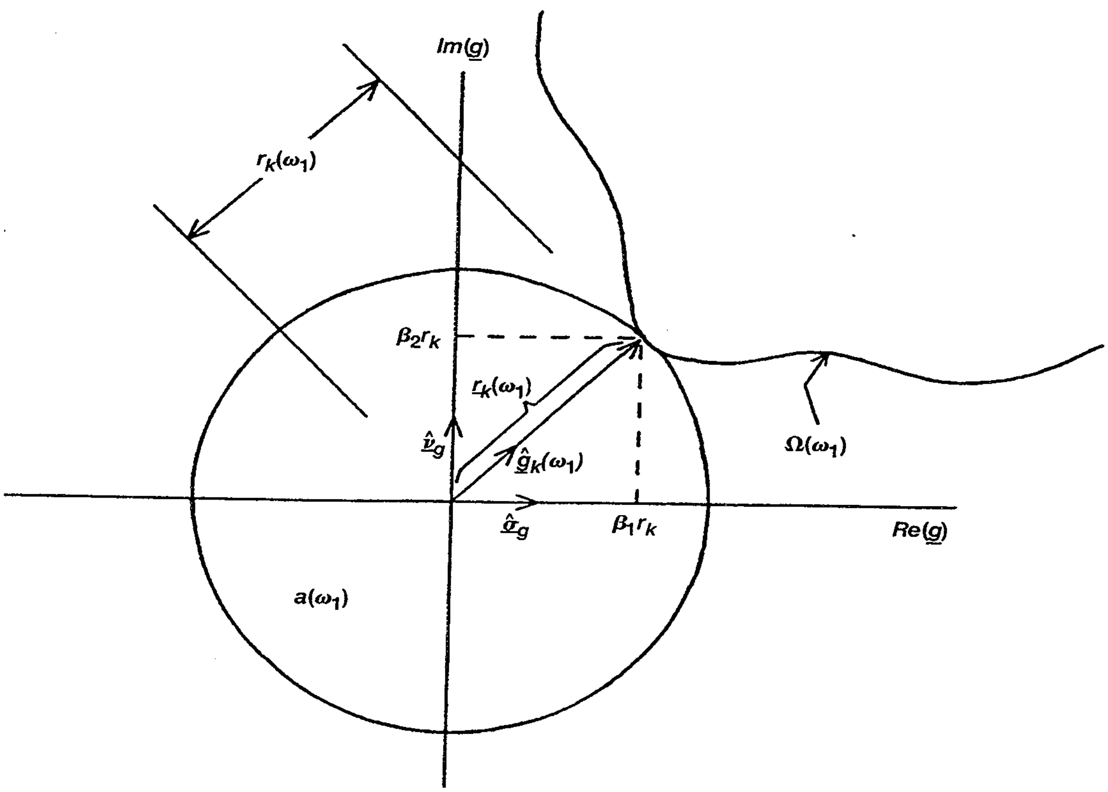

Figure 12b.—Complex feedback uncertainty acceptable region for a particular frequency ω_1.

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Many acceleration-sensitive, microgravity science experiments will require active vibration isolation from the manned orbiters on which they will be mounted. The isolation problem, especially in the case of a tethered payload, is a complex three-dimensional one that is best suited to modern-control design methods. These methods, although more powerful than their classical counterparts, can nonetheless go only so far in meeting the design requirements for practical systems. Once a tentative controller design is available, it must still be evaluated to determine whether or not it is fully acceptable, and to compare it with other possible design candidates. Realistically, such evaluation will be an inherent part of a necessarily iterative design process. In this paper, an approach is presented for applying complex μ-analysis methods to a closed-loop vibration isolation system (experiment plus controller). An analysis framework is presented for evaluating nominal stability, nominal performance, robust stability, and robust performance of active microgravity isolation systems, with emphasis on the effective use of μ-analysis methods.

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